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Possible quantum gravity effects in a charged Bose condensate under variable e.m. field.

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Abstract

We prove that in Euclidean quantum gravity, in the weak field approximation, a "local" positive cosmological term $\mu^2(x)$ induces localized gravitational instabilities. Such a term can be produced by the coupling to an external Bose condensate. We study the static classical limit of the functional integral in the presence of the (regulated) instabilities.

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This model is applied to a phenomenological analysis of recent experimental results. A new demonstration experiment is described.

04.20.-q Classical general relativity.

04.60.-m Quantum gravity.

74.72.-h High-T_c cuprates.

The behavior of a Bose condensate – or more specifically of a superconductor – in an external gravitational field has been the subject of some study in the past [1]. The presence in a superconductor of currents flowing without any measurable resistance suggests that it could be used as a sensitive detection system, in particular for gravitational fields. The possible back-reaction of induced supercurrents on the gravitational field itself has been studied too, in analogy with the familiar treatment of the Meissner effect. As one can easily foresee, it turns out that the "gravitational Meissner effect" is extremely weak: it was computed for instance that in a neutron star with density of the order of $10^{17}\ kg/m^3$ the London penetration depth is ca. 12 km [2].

The reason for the extremely weak coupling of the supercurrents to the classical gravitational field is very general and originates of course from the smallness of the coupling between gravity and the energy-momentum tensor of matter $T_{\mu\nu}$. One might wonder whether in a quantum theory of gravity – or at least in an approximation of the theory for weak fields – the Bose condensate of the Cooper pairs, due to its macroscopic quantum character, can play a more subtle role than a simple contribution to the energy-momentum tensor.

In a quantum-field representation the condensate is described by a field with non-vanishing vacuum expectation value ϕ_0, possibly depending on the spacetime coordinate x. It is interesting to insert the action of this field, suitably "covariantized", into the functional integral of gravity, expand the metric tensor $g_{\mu\nu}$ in weak field approximation and check if the only effect of $\phi_0(x)$ is to produce gravity/condensate interaction vertices proportional to powers of $\kappa = \sqrt{16\pi G}$. One finds that in general this is not the case; the quadratic part of the gravitational action is modified too, by receiving a negative definite contribution. It can thus be expected that the condensate induces localized instabilities of the gravitational field, in a sense which we shall precise in Section 1.

The present paper is based on the letter [5] and originates in part from previous theoretical work [3] and in part from recent experimental results ([4, 21]; see Section 2) which show the possibility of an anomalous interaction, in special conditions, between the gravitational field and a superconductor. We have developed a theoretical model (Section 3) which on the basis of the general results of Section 1 allows to interpret in a consistent way the main reported experimental observations. In Section 4 we collect some general considerations concerning the total energetic balance of the process described in Section 3. In Section 5 we analyze arguments for and against the hypothesis of a "threshold density" for the condensate density and finally Section 6 comprises some conclusive remarks. Sections 2, 3 (in part) and 4 have a less complex formal content than Section 1 and are readable without a detailed knowledge of quantum field theory.

1 Effect of a local cosmological term in Euclidean quantum gravity.

1.1 Global cosmological term.

Let us consider the action of the gravitational field $g_{\mu\nu}(x)$ in its usual form:

$$S_g = \int d^4x \sqrt{g(x)} \left[\frac{\Lambda}{8\pi G} - \frac{1}{8\pi G} R(x) \right], \tag{1}$$

where $-R(x)/8\pi G$ is the Einstein term and $\Lambda/8\pi G$ is the cosmological term which generally can be present.

It is known that the coupling of the gravitational field with another field is formally obtained by "covariantizing" the action of the latter; this means that the contractions of the Lorentz indices of the field must be effected through the metric $g_{\mu\nu}(x)$ or its inverse $g^{\mu\nu}(x)$ and that the ordinary derivatives are transformed into covariant derivatives by inserting the connection field. Moreover, the Minkowskian volume element d^4x is replaced by $d^4x \sqrt{g(x)}$, where $g(x)$ is the determinant of the metric. The insertion of the factor $\sqrt{g(x)}$ into the volume element has the effect that any additive constant in the Lagrangian contributes to the cosmological term $\Lambda/8\pi G$. For instance, let us consider a Bose condensate described by a scalar field $\phi(x) = \phi_0 + \hat{\phi}(x)$, where ϕ_0 is the vacuum expectation value and $m_\phi|\phi_0|^2$ represents the particles density of the ground state in the non-relativistic limit (compare Section 5). The action of this field in the presence of gravity is

$$S_\phi = \frac{1}{2}\int d^4x\,\sqrt{g(x)}\left\{[\partial_\mu\hat{\phi}(x)]^*[\partial_\nu\hat{\phi}(x)]g^{\mu\nu}(x) + m_\phi^2|\hat{\phi}(x)|^2 + m_\phi^2[\phi_0^*\hat{\phi}(x) + \hat{\phi}^*(x)\phi_0] + m_\phi^2|\phi_0|^2\right\}. \tag{2}$$

One can easily check that in the total action $(S_g + S_\phi)$ the contribution $\frac{1}{2}m_\phi^2|\phi_0|^2 8\pi G$ is added to the "intrinsic" gravitational cosmological constant Λ. (Note that in the covariantized action above the derivatives are unchanged, since $\hat{\phi}(x)$ is a scalar quantity.)

The astronomical observations impose a very low limit on the total cosmological term present in the action of the gravitational field. The presently accepted limit is of the order of $|\Lambda|G < 10^{-120}$, which means approximately for Λ itself $|\Lambda| < 10^{-54}\ cm^{-2}$ (we use natural units, in which $\hbar = c = 1$ and thus Λ has dimensions cm^{-2}, while G has dimensions cm^2; $G \sim L_{Planck}^2 \sim 10^{-66}\ cm^2$). This absence of curvature in the universe at large scale rises a paradox, called "the cosmological constant problem" (see [6]). In fact the Higgs fields of the standard model as well as the zero-point fluctuations of any quantum field including the gravitational field itself generate huge contributions to the cosmological term, which however appear to be somehow "rescaled" to zero at macroscopic distances. In order to explain how this can occur, several quantum field theoretical models have been proposed [7]. No definitive and universally accepted solution of the paradox seems to be at hand yet, since that would require in fact a complete non-perturbative treatment of gravity which appears not feasible up to now.

A model in which the large scale vanishing of the effective cosmological constant is reproduced in a natural way through numerical simulations is the Euclidean quantum gravity on the Regge lattice [8]. From this model emerges a property, which could turn out to be more general than the model itself: if we keep the fundamental length L_{Planck} in the theory, the vanishing of the effective cosmological constant $|\Lambda|$ in dependence of the energy scale p follows a law of the form $|\Lambda|(p) \sim G^{-1}(L_{Planck}\,p)^\gamma$, where γ is a critical exponent [9, 11]. It is not excluded that this behavior of the effective cosmological constant may be observed in certain circumstances (see

[12] and Section 5). Furthermore, the model predicts that in the large distance limit Λ goes to zero while keeping negative sign. Also this property has probably a more general character, since the weak field approximation for the gravitational field is "stable" in the presence of an infinitesimal cosmological term with negative sign, while on the contrary it becomes unstable in the presence of a positive cosmological term (see Section 1.5).

1.2 Local cosmological term.

Summarizing, independently of the model there appears to exist a dynamical mechanism which "rescales to zero" any contribution to the cosmological term and fortunately makes the gravitational field insensitive to any constant term in the action of other fields coupled to it. Nevertheless, let us go back to the previously mentioned example of a Bose condensate described by a scalar field $\phi(x) = \phi_0 + \hat{\phi}(x)$. If the vacuum expectation value ϕ_0 is not constant but depends on the spacetime coordinate x, in the gravitational action S_g appears a positive "local" cosmological term which can have interesting consequences. Let us suppose that $\phi_0(x)$ is fixed by external factors and let us decompose the gravitational field $g_{\mu\nu}(x)$ as usual in the weak field approximation, that is, $g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x)$, where $\kappa = \sqrt{8\pi G} \sim L_{Planck}$. The total action of the system takes the form

$$S = \int d^4x \sqrt{g(x)} \left\{ \left[\frac{\Lambda}{8\pi G} + \frac{1}{2}\mu^2(x) \right] - \frac{1}{8\pi G} R(x) \right\} + S_{h\phi_0} + S_{\hat{\phi}}, \tag{3}$$

where

$$\frac{1}{2}\mu^2(x) = \frac{1}{2}[\partial_\mu \phi_0^*(x)][\partial^\mu \phi_0(x)] + \frac{1}{2} m_\phi^2 |\phi_0(x)|^2; \tag{4}$$

$$S_{h\phi_0} = \frac{1}{2} \int d^4x \sqrt{g(x)}\, \partial^\mu \phi_0^*(x) \partial^\nu \phi_0(x) \kappa h_{\mu\nu}(x); \tag{5}$$

$$\begin{aligned} S_{\hat{\phi}} = \frac{1}{2} \int d^4x \sqrt{g(x)} \Big\{ & m_\phi^2 |\hat{\phi}(x)|^2 + m_\phi^2 \left[\phi_0^*(x)\hat{\phi}(x) + \phi_0(x)\hat{\phi}^*(x) \right] + \\ & + \left[\partial_\mu \hat{\phi}^*(x) \partial_\nu \hat{\phi}(x) + \partial_\mu \phi_0^*(x) \partial_\nu \hat{\phi}(x) + \partial_\mu \hat{\phi}^*(x) \partial_\nu \phi_0(x) \right] g^{\mu\nu}(x) \Big\}. \end{aligned} \tag{6}$$

In the action above the terms $S_{h\phi_0}$ and $S_{\hat{\phi}}$ represent effects of secondary importance. The term $S_{h\phi_0}$ describes a process in which gravitons are produced by the "source" $\phi_0(x)$. The term $S_{\hat{\phi}}$ contains the free action of the field $\hat{\phi}(x)$ describing the excitations of the condensate, and several vertices in which the graviton field $h_{\mu\nu}(x)$ and $\hat{\phi}(x)$ interact between themselves and possibly with the source. All these interactions are not of special interest here and are generally very weak, due to the smallness of the coupling κ. The relevant point (eq.s (3), (4)) is that the purely gravitational cosmological term $\frac{\Lambda}{8\pi G}$ receives a local positive contribution $\frac{1}{2}\mu^2(x)$ which depends on the external source $\phi_0(x)$.

We shall call "critical regions" the regions of spacetime in which the following condition is satisfied:

$$\left[\frac{\Lambda}{8\pi G} + \frac{1}{2}\mu^2(x)\right] > 0. \tag{7}$$

Since we know that the intrinsic cosmological term $\Lambda/8\pi G$ is very small, we expect that these regions are essentially determined by the source term $\mu^2(x)$ and thus, through (4), by the vacuum expectation value $\phi_0(x)$. We shall discuss later (Section 5) whether there might be a competition between the two terms in (7) and therefore a "threshold" effect.

1.3 Euclidean theory for weak fields.

It is more convenient at this point to carry on our analysis in the Euclidean formalism, in which the Minkowski metric $\eta_{\mu\nu}$ is replaced by the four-dimensional Euclidean metric $\delta_{\mu\nu}$. This amounts to replace the time variable with an imaginary variable and requires that the theory behaves regularly with respect to a rotation of the time axis in the complex plane. For the familiar quantum field theories in flat spacetime this requirement is usually satisfied, but in the case of quantum gravity the situation is in general much more complicated since the metric itself belongs to the dynamic variables of the theory. The equivalence of the gravitational Euclidean theory [13] with the theory in real spacetime has not been proved yet, in spite of the considerable efforts in this direction [14]. Anyway, such an equivalence would be mainly formal, as neither theory is well defined in a general sense.

We should also mention that in general the Euclidean Einstein action (the term $\frac{-1}{8\pi G}\int \sqrt{g}R$ in eq. (1)) is not bounded from below [13]. Thus it is not possible to obtain the vacuum state of the quantum theory (flat space) by minimizing the action in an elementary way, like in the usual Euclidean theories. Several solutions to this problem have been proposed which exploit the freedom in the choice of the functional integration measure, the stochastic regularization or the regularization through an R^2 term, effective only at very small distances †.

Nevertheless, since in our case the gravitational field is always regarded as weak (small fluctuations around a flat background), it is not necessary in fact to use Euclidean quantum gravity in its most general form. We may treat it, "*a la* particle physics", like a normal quantum field theory (or possibly an effective low-energy theory [15]) in which the background metric is fixed and the analytical continuation between the Euclidean and Minkowskian case is well defined. According to this approximation and to the physical reality we shall suppose that in the absence of external sources the ground state of gravity is flat spacetime, at least at macroscopic scale. We do not specify the dynamical mechanism through which this ground state emerges from the complete theory, although we regard the mentioned non-perturbative Regge calculus simulations as particularly instructive in this sense.

The quadratic part of the Euclidean Einstein action is positive-definite on the average (see Section 1.4) and thus effectively stable with respect to weak fluctuations. In any case, as we shall see, we are not really interested here in the stability of the R-term in the action, but into that of a term of the form $\Lambda_{eff}\sqrt{g}$ (see eq. (14)). If necessary we can admit that the Euclidean Einstein action has been suitably modified in order to allow a correct analytical continuation and to make it bounded from below also beyond the weak field approximation (compare [14] and references) and this will not affect our conclusions.

1.4 Quadratic part of the action in harmonic gauge.

Let us consider the Einstein action with cosmological term (1) in the approximation of a weak Euclidean field $g_{\mu\nu}(x) = \delta_{\mu\nu} + \kappa h_{\mu\nu}(x)$, where $\kappa = \sqrt{8\pi G}$. We first observe [16] that after the addition of a harmonic gauge-fixing the quadratic part of the action takes the form

$$S_g^{(2)} = \int d^4x\, h_{\mu\nu}(x) V^{\mu\nu\alpha\beta}(-\partial^2 - \Lambda) h_{\alpha\beta}(x), \tag{8}$$

† See for instance [8, 14]. The R^2 term makes the theory renormalizable but non-unitary at perturbative level. However, it is believed that the unitarity problem should be solved by the complete theory.

where $V^{\mu\nu\alpha\beta} = \delta^{\mu\alpha}\delta^{\nu\beta} + \delta^{\mu\beta}\delta^{\nu\alpha} - \delta^{\mu\nu}\delta^{\alpha\beta}$. In momentum space (8) becomes

$$
\begin{aligned}
S_g^{(2)} &= \int d^4p\, \tilde{h}^*_{\mu\nu}(p) V^{\mu\nu\alpha\beta}(p^2 - \Lambda)\tilde{h}_{\alpha\beta}(p) && (9) \\
&= \int d^4p\, (p^2 - \Lambda)\left\{2\mathrm{Tr}[\tilde{h}^*(p)\tilde{h}(p)] - |\mathrm{Tr}\tilde{h}(p)|^2\right\}. && (10)
\end{aligned}
$$

From (9) one often desumes that in this approximation a cosmological term amounts to a mass term for the graviton, positive if $\Lambda < 0$ and negative (with consequent instability of the theory; see also [12] and references therein) if $\Lambda > 0$. Since in the presence of a condensate the sign of the total cosmological term depends on the coordinate x (eq. (3)) we might expect some "local" instabilities in the critical regions (7) in that case.

We make now a short digression from our main argument and check the positivity of the quadratic part of the Euclidean Einstein action in harmonic gauge. Note that since h is a symmetric tensor, we can diagonalize it at any point before extracting its trace. The quadratic form $2\mathrm{Tr}(\tilde{h}^*\tilde{h}) - |\mathrm{Tr}\tilde{h}|^2$ in (10), expressed in terms of the diagonal $\tilde{h}$, namely

$$
Q = 2\sum_\alpha |\tilde{h}_{\alpha\alpha}|^2 - |\sum_\alpha \tilde{h}_{\alpha\alpha}|^2 = \sum_\alpha |\tilde{h}_{\alpha\alpha}|^2 - \sum_{\alpha\neq\beta} \tilde{h}^*_{\alpha\alpha}\tilde{h}_{\beta\beta} \qquad (11)
$$

has negative as well as positive eigenvalues. This should be expected, as the scalar curvature R has no definite sign, and means that the quadratic part of the action has no minimum for $h = 0$. Remember however that we are working in the Euclidean functional formulation of the theory, so the field h is supposed to "fluctuate thermally" with temperature $\Theta = \hbar/k_B$ (compare also eq.s (22), (23)). Also note that having already diagonalized h at any point, we cannot do any further linear transformation on it. Taking the mean value of Q and assuming that the correlations $\langle h_{\alpha\alpha}h_{\beta\beta}\rangle$ vanish by isotropy for $\alpha \neq \beta$ we obtain $\langle Q\rangle = \sum_\alpha \langle |h_{\alpha\alpha}|^2\rangle$. We conclude that the quadratic part of the Einstein action is stable "on the average" at $h = 0$. For an effective theory (compare our discussion in Section 1.3) this should be sufficient.

1.5 Quadratic part of $g^{1/2}$ and instabilities.

We can prove the appearance of instabilities in the presence of a positive cosmological term at a more general level. Before introducing any gauge-fixing for the gravitational field and disregarding the Einstein action for a moment, let us study the stability of the cosmological term $\frac{1}{8\pi G}\sqrt{g}$. The expansion of the determinant g gives to first order in κ

$$g^{(1)} = \kappa \mathrm{Tr} h \tag{12}$$

and to second order

$$g^{(2)} = \frac{1}{2}\kappa^2 \left[(\mathrm{Tr} h)^2 - \mathrm{Tr}(h^2)\right]. \tag{13}$$

Recalling the expansion $\sqrt{1+\delta} = 1 + \frac{1}{2}\delta - \frac{1}{8}\delta^2 + ...$ and rearranging the second order terms one finds

$$[\sqrt{g}]^{(2)} = \frac{1}{8}\kappa^2 \left[(\mathrm{Tr} h)^2 - 2\mathrm{Tr}(h^2)\right]. \tag{14}$$

(By the way, we have checked in this fashion that the quadratic part of $\sqrt{g(x)}$ has the same algebraic structure in h as the Einstein term in harmonic gauge.)

Now we can impose the condition $\mathrm{Tr} h = 0$ [‡] and look at the stability of the total action $\frac{1}{8\pi G}\int d^4x\, \sqrt{g}(\Lambda_{eff} - R)$. If $\Lambda_{eff} > 0$ the cosmological term is clearly unbounded from below with respect to any "zero mode" $h(x)$ which leaves unchanged the rest of the Euclidean action – that is, the Einstein term $\frac{-1}{8\pi G}\int d^4x \sqrt{g} R$. This requirement is satisfied by fields h which are solutions of Einstein equations

$$R_{\mu\nu}(x) - \frac{1}{2} g_{\mu\nu}(x) R(x) = -8\pi G T_{\mu\nu}(x) \tag{18}$$

with $T_{\mu\nu}$ satisfying the condition

$$\int d^4x \sqrt{g(x)}\, \mathrm{Tr}\, T(x) = 0 \tag{19}$$

‡ We can always impose $\mathrm{Tr} h = 0$ in the Euclidean weak field approximation, since the gauge transformations have the form

$$h_{\mu\nu}(x) \to h_{\mu\nu}(x) + \partial_\mu f_\nu(x) + \partial_\nu f_\mu(x) \tag{15}$$

with $f_\mu(x)$ arbitrary and thus the condition on the transformation is

$$\partial^\mu f_\mu(x) = -\frac{1}{2}\mathrm{Tr} h(x). \tag{16}$$

By choosing $f_\mu(x)$ of the form $\partial_\mu F(x)$ we obtain the condition

$$\partial^2 F(x) = -\frac{1}{2}\mathrm{Tr} h(x), \tag{17}$$

which in Euclidean spacetime is easily solved.

(note that for solutions of (18) one has $R(x) = 8\pi G \,\mathrm{Tr}\, T(x)$).

In Minkowski space one can easily exhibit zero modes of Einstein action: namely gravitational waves satisfy eq. (18) with $T_{\mu\nu}(x) = 0$ and in particular they satisfy a linear wave equation in the weak-field approximation. In Euclidean space the linearized Einstein vacuum equations take the form of four-dimensional Poisson equations, as can be seen most easily in harmonic gauge. Thus they do not admit nontrivial solutions for $T(x) = 0$ everywhere. However condition (19) can be also satisfied by energy-momentum tensors which are not identically zero but may have negative and positive sign, in such a way that their total integral is zero. Of course, they do not represent any acceptable physical source, but the corresponding solutions of (18) exist anyway and are zero modes of the Euclidean action. One can consider, for instance, the static field produced by a "mass dipole" (which eventually we imagine as centered in a critical region), with behaviour $h \sim r^{-2}$, etc.

In conclusion, in the regions where $\Lambda_{eff} > 0$ the zero modes of $h_{\mu\nu}(x)$ tend to grow without restriction. In the case of the interaction with a Bose condensate, such regions are the critical regions (7). Since we are considering a weak-field approximation, we shall assume that in fact in those regions the field oscillates between extremal values, with null average. With an expression borrowed from experimental physics, we might say that h^2 "saturates" in those regions. The extremal values will be determined by some physical cut-off and are not relevant if we are concerned only with the average (compare Section 3).

1.6 Comparison with the classical case.

The instability effect described above is of quantum nature. In General Relativity the consequences of a positive effective cosmological term $\Lambda_{eff}(x)$ are not quantitatively different from those of an ordinary mass-energy density $T_{00}(x)$ and we do not see any hint of instability in the corresponding solutions of classical Einstein equations. For instance, the trace of the Einstein vacuum equations derived from the action (1) is $R(x) = \Lambda_{eff}(x)$. On the other hand, taking the trace of the equations without cosmological term but in the presence of matter we obtain as mentioned

$$R(x) = 8\pi G \,\mathrm{Tr}\, T(x). \tag{20}$$

This means that in the regions in which $\Lambda_{eff}(x)$ or $T_{00}(x)$ are different from zero, the curvature radius ρ of spacetime is of the order of $\rho \sim 1/\sqrt{\Lambda_{eff}(x)}$ or $\rho \sim 1/\sqrt{T_{00}(x)}$. For an ordinary density ρ is very large, say $\rho \sim 10^{16}$ cm at least (see Section 5).

One key difference between a classical "geometrodynamical" view of General Relativity and the quantum field theory on a flat background lies in the interpretation of the factor $\sqrt{g}$ in the action. In order to derive the classical equations (18) from the total action of the [gravity+matter] system one defines the energy-momentum tensor $T^{\mu\nu}(x)$ in such a way that a variation $\delta g_{\mu\nu}(x)$ of the metric affects the action as follows:

$$\delta S_{matter} = \frac{1}{2} \int d^4x \, \sqrt{g(x)} \, T^{\mu\nu}(x) \delta g_{\mu\nu}(x). \tag{21}$$

In the quantum theory of gravity on a flat background "*a la* particle physics", which we are adopting, the point of view is different. The factor $\sqrt{g(x)}$ does not have the classical geometrical meaning of four-volume measure anymore. It is expanded in a power series of κ and the contributions obtained in this way are added to the rest of the covariantized action, giving rise as we saw also to a negative quadratic term in h (eq. (14)). Thus the instability which follows from this term for $\Lambda_{eff} > 0$ should be regarded as a quantum gravity effect, connected to the perturbative theory around a flat background.

Finally we recall that in several cosmological theories [17] one assumes that at some stage of the evolution of the universe there can be large negative contributions to T^{00}, due to the instability of certain fields or to the formation of a condensate. Our model does not have anything in common with these (essentially classical) theories.

1.7 General functional integral for the static potential.

In the following we shall be interested in the influence of the induced cosmological term $\mu^2(x)$ on the gravitational interaction of two masses m_1 and m_2 at rest. This influence can be computed in principle inserting $\mu^2(x)$ in the general formula for the static potential in Euclidean quantum gravity [3, 9, 10]

$$U(L) = \lim_{T \to \infty} -\frac{\hbar}{T} \log \frac{1}{Z} \int d[g] \, \exp \left\{ -\hbar^{-1} \left[S_g + \sum_{i=1,2} m_i \int_{-\frac{T}{2}}^{\frac{T}{2}} dt \, \sqrt{g_{\mu\nu}[x_i(t)] \dot{x}_i^\mu(t) \dot{x}_i^\nu(t)} \right] \right\} \tag{22}$$

where S_g is the gravitational action (1) and Z a normalization factor. The trajectories $x_i(t)$ of the two masses m_1 and m_2 are parallel with respect to the metric g. L is the distance between the trajectories, corresponding to the spatial distance of the two masses. The interaction energy $U(L)$ of the two masses depends on the correlations between the values of the gravitational field on the "Wilson lines" $x_1(t)$ and $x_2(t)$. This can be verified explicitly in the weak field approximation or through numerical simulations.

We can rewrite eq. (22) in the presence of the Bose condensate as

$$\begin{aligned} U[L, \mu^2(x)] &= \lim_{T\to\infty} -\frac{\hbar}{T} \log \frac{1}{Z} \int d[g] \int d[\hat{\phi}] \\ & \exp\left\{ -\hbar^{-1} \left[S[g, \hat{\phi}, \phi_0] + \sum_{i=1,2} m_i \int_{-\frac{T}{2}}^{\frac{T}{2}} dt \sqrt{g_{\mu\nu}[x_i(t)]\dot{x}_i^\mu(t)\dot{x}_i^\nu(t)} \right] \right\}, \end{aligned} \quad (23)$$

where $S[g, \hat{\phi}, \phi_0]$ is the total action defined in (3). We have seen that in the weak field approximation h^2 "saturates" in the critical regions. As a consequence, the field correlations present in the functional integral are modified. We showed earlier with a simple numerical model [12] that in general this reduces $|U|$. Also from an intuitive point of view it is quite clear that local constraints on the field damp its correlations. An explicit evaluation of the functional integral (23) is however quite difficult. In Section 3 we shall work out a simpler model based on a classical limit.

Two final remarks are in order:

1. Notice that according to the general definition of the center of mass of a system in the presence of gravity eq. (22) holds, more generally, when $x_i(t)$ represent the trajectories of the centers of mass of two extended bodies. This property can explain why the observed height dependence of the shielding effect is so weak (compare Section 2 and [18, 20]) although the disk apparently should "shield only part of the Earth".

2. We observe that in order to affect the quadratic part of the gravitational action the local cosmological term $\mu^2(x)$ must contain only fields, like $\phi_0(x)$, which do not belong to the functional integration variables. For instance, the terms containing $\hat{\phi}(x)$ in eq. (6) are not included in $\mu^2(x)$ (eq. (4)), because in perturbation theory they represent simply

interaction vertices. This remains true even if $\hat{\phi}(x)$ is coupled in turn to a further external source.

This reasoning can be generalized to other fields possibly present in the total action. One concludes that a cosmological contribution like $\mu^2(x)$ can only originate from a field with non-vanishing vacuum expectation value. In our case the vacuum expectation value $\phi_0(x)$ is the result of a number of external physical factors: the action of the e.m. field (see Section 2), the equilibrium of the thermodinamic potentials of the condensate in the given conditions of temperature, the microscopic structure of the superconducting material etc.

2 Experimental evidences.

2.1 Summary.

A recent experiment [4] has shown an unexpected interaction between the gravitational field and a superconductor subjected to external e.m. fields. In this Section we summarize the main reported observations, trying to focus on the essential elements, since the experimental situation is quite complex. We add a few qualitative remarks on the possible theoretical interpretations of these observations according to the model with "anomalous" coupling between $h(x)$ and $\phi_0(x)$ introduced in the previous Sections. A more quantitative application of the model is given in Section 3.

The core of the experimental apparatus is a toroidal disk of diameter 27 cm made of high critical temperature (HTC) superconducting material. The disk is kept at a temperature below 70 K; it levitates above three electromagnets and rotates (up to ca. 5000 rpm) due to the action of additional lateral magnetic fields. All electromagnets are supplied with AC current with variable frequency. Within certain frequency ranges one observes a slight decrease in the weight of samples hung above the disk, up to a maximum "shielding" value of ca. 1%. A smaller effect, of the order of 0.1% or less, is observed if the disk is only levitating but not rotating.

The percentage of weight decrease is the same for samples of different masses and chemical compositions. One can thus describe the effect as a slight diminution of the gravity acceleration

g_E above the disk. The dependence of the effect on the height above the disk is very weak: there appears to be in practice a "shielding cylinder" over the disk (compare also [18, 20]), extending for 3 meters at least. The resulting field configuration is clearly non conservative. An horizontal force at the border of the shielding cylinder has occasionally been observed, but it is far too small to restore the usual "zero circuitation" property of the static field. No weight reduction is observed under the disk.

The disk has a composite microscopic structure: the upper part is treated with a thermal process which melts partially the grains of the HTC material, while the lower part is more granular and has a lower critical temperature. This double structure aims at obtaining good levitation properties of the disk, while leaving also a layer in which considerable resistive effects can arise. In general both requirements appear necessary for the effect to take place: (1) the disk must be able to support intense super-currents; (2) a granular structure with pinning centers must be present, such to oppose resistance to variations of the super-currents pattern while the disk is subjected to alternate e.m. fields.

2.2 Interpretation.

We have already stressed in our analysis [5] that an interpretation of the reported effect in the framework of General Relativity, as due to repulsive post-newtonian fields produced by the super-currents [19], is untenable, since the magnitude order of the effect is far too large. The work [20] shows that the contribution from the super-currents to the static component g_{00} of the post-newtonian gravitational field over the disk is not only much smaller than the observed effect (by several magnitude orders), but it is attractive like the newtonian field of the Earth. Even taking into account perturbative quantum corrections to the Newton potential one reaches the same negative conclusion.

Our interpretative model of the experimental results [5] is based on the "anomalous" coupling between Bose condensate and gravitational field described by eq.s (3), (4). In this model the essential ingredient for the shielding is the presence of strong variations of the Cooper pairs density in the disk: *we assume that such variations produce small regions with higher density, where the criticality condition (7) is satisfied and thus an additional boundary condition* *is imposed on the gravitational field.* [§]

In this view the disk's rotation in the external magnetic field plays the role of forcing the pattern of the super-currents and thus the local condensate density. It is known that in general by moving a type II superconductor (of which the HTC are one example, that is, with a structure of superconducting and non-superconducting regions) in an external field or by applying to it an AC, one causes resistive phenomena, since the super-currents' pattern is unable to follow the fields variations. In the present case it has been in fact observed that while the peak shielding values are produced, the disk tends to heat up.

This point of view – suggested by a long analysis of the experimental results in search of a consistent interpretation – allows one to regard the experimental conditions reported in [4] as a particular case, open to changes and simplifications. The levitation of the disk does not appear to be *per se* necessary, but just convenient in order to rotate it. In turn, the rotation in the external field aims essentially, as mentioned, at forcing the currents' pattern. Thus one might perhaps simply rotate the disk mechanically in a fixed external field, or rapidly vary the fields direction and strength while leaving the disk at rest. The latter technique has been recently employed with positive results (Section 2.3).

Even if we stick to the relatively simple picture above, according to which the essential are the variations of the Cooper pairs density, there remains for the theory the general task of predicting such variations in dependence of the disk structure, of the external fields etc. This is clearly a very difficult task, especially for an HTC superconductor. It seems more likely at present that the optimization of the parameters mentioned above (disk structure, external fields, etc.) will be first approached in a semi-empirical way.

Another crucial feature of the experimental apparatus is the frequency spectrum of the applied e.m. field. Independently of the reason for which the external e.m. field was originally employed, it is clear in our opinion that it plays a fundamental role in supplying the energy necessary for the "absorption" of the gravitational field in the critical regions. A simple model

[§] Clearly the local condensate density can never exceed the total electronic density. The average condensate density also increases at lower temperatures but the power transfer process becomes more difficult in that case (compare Section 5.2).

which describes this mechanism is presented in Sections 3, 4. Experimentally one observes [4] that the maximum shielding value is obtained when the coils are supplied with high frequency current (of the order of 10 MHz).

In general, the power transfer to the disk has necessarily a limited efficiency. This represents one of the most serious problems to overcome in order to obtain the shielding effect in stable form, especially for heavy samples, without using an excessive amount of refrigerating fluid to avoid the heating of the disk and the ensuing loss of its superconducting properties. One should also remind that the maximum shielding value was observed in conditions close to the resistive transition. This makes clear why it is important to use a disk made of HTC material: in a low-temperature superconductor, admitted one can reach the critical density conditions, the specific heat is probably too small to maintain them in a stable way and to allow any power transfer.

2.3 A new demonstration experiment.

One of us (J.S.) has recently succeded [21] in reproducing the weak gravitational shielding effect for a short time interval (up to 5 seconds). The experimental setup was designed in such a way to eliminate as far as possible any non-gravitational disturbance and to show a precise temporal correspondence between actions taken on the HTC disk and the weight reduction of the samples. Although the observed weight reduction was quite large (of the order of 5%) this experiment should be regarded just as a demonstration experiment. In fact, many actions had to be taken literally by hand, the samples employed were in all cases very light and the short duration of the effect did not allow any precise spatial mapping of the field.

It is very remarkable that in this experiment the effect was obtained without subjecting the disk to any rotation. This supports our interpretation of the effect and rises hopes that the original experimental setup of Podkletnov and co-workers could be substantially simplified. However, at the present there is no evidence that the effect can be obtained in stable form without rotation ¶. Also, it should be mentioned that similar temporary effects were observed

¶ According to public releases, the NASA group in Huntsville, Alabama, is cloning Podkletnov's experiment. This is a difficult task, especially for the sophisticated technology involved in the construction of the large HTC by Podkletnov et al. since the first measurements, too [4].

The experimental setup consists of (a) a hexagon-shaped YBCO 1" (2.5 *cm*) superconducting disk, 6 *mm* thick; (b) a magnetic field generator producing a 600 *gauss*/60 *Hz* e.m. field; (c) a beam balance with suspended sample.

The beam is made of bamboo, without any metal part, coming to a point on one end (24.6 *cm* long, weight 1.865 *g*). The sample is made as follows. A cardboard rectangle (16 *mm* by 10 *mm* by 0.13 *mm*) is suspended from the balance with 2.8 *cm* of cotton string. A polystyrene "pan" (7.2 *cm* by 8.7 *cm* by 1.7 *mm*) is attached with paper masking tape to the cardboard rectangle. The total sample assembly (with string, cardboard, tape) weighs 1.650 *g*.

The balance is suspended from the end of a 150 *cm* wood crossbeam by ca. 30 *cm* of monofilament fishing line (8 lb. test) attached to the balance's center of mass (5.5 *cm* from the end where the sample is attached). The other end of the crossbeam is firmly anchored by a heavy steel tripod. Thermal and e.m. isolation is provided by a glass plate (15 *cm* by 30 *cm*, 0.7 *cm* thick) with a brass screen attachment. This plate-and-brass-screen assembly is held about 4.5 *cm* below the sample by a "3-finger" ring stand clamp. A straightsided, 6" diameter, 10" deep dewar with 3-4" of liquid nitrogen is used to cool the superconducting disk below its critical temperature, and is removed from the experiment area before the trial.

The experimental procedure comprises the following steps.

1. The YBCO superconductor is placed in a liquid nitrogen bath and allowed to come to liquid nitrogen temperature (as indicated when the boiling of the liquid nitrogen ceases). The superconductor will remain below its critical temperature (about 90 K) for the duration of the trial (less than 20 seconds).

2. The disk is then removed from the bath and placed on a strong NdFeB magnet to induce a supercurrent. The Meissner effect is counteracted by a wooden stick. The superconducting disk has a cotton string attached to it to assist handling.

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disk and in the control of its rotation. We are also aware, though still at un-official level, of other groups working at the experiment with smaller disks.

3. The disk and wooden stick assembly is placed on the AC field generator, about 33 *cm* below the isolation plate and about 40 *cm* below the sample. The AC field generator is then cycled for ca. 5 seconds with 0.75 *sec* equal-time on/off pulses. Prior to a run the sample is centered to be over the middle of where the disk will finally be, on the AC field generator. The idea is that the "column" of modified gravity has to hit the sample somewhere as the disk is only 1 inch in diameter and the sample is much larger.

One observes that while AC current is flowing through the generator the balance pointer dips 2.1 *mm* downward. When the AC field generator is pulsed with no superconductor present, there is no measurable pointer deflection. Also air flows do not cause any measurable deflection. The whole procedure is well reproducible.

The weight difference required to raise the sample by 2.1 *mm* was then found to be 0.089 *g*. This was measured taking advantage of the fact that the suspension wire produces a small torque on the balance beam toward the equilibrium position: the balance pointer was found to raise 2.1 *mm* upward when a weight of 0.089 *g* was placed above the sample.

An improved version of the experiment is being developed. Details will appear elsewhere.

3 The modified field to lowest order.

3.1 Static classical limit of the functional integral.

In this Section we study in a suitable approximation the consequences of the anomalous coupling between the gravitational field and the Bose condensate taking place in the critical regions (i.e., where condition (7) is satisfied). We aim at verifying in this way whether from our theoretical hypoteses follow plausible phenomenological consequences and at composing a picture of the shielding phenomenon which may help in understanding various aspects: in which sense a constant gravitational field is slightly "absorbed" in the disk and turns out to be weaker above it; if the field modified in this way is conservative; which is the global energy balance of the process etc.

We have seen that in the presence of strong variable e.m. fields, in the superconducting disk small regions can appear in which the Cooper pairs density is particularly high. We shall discuss later whether there is necessarily a "threshold" density which must be exceeded for the anomalous coupling to be possible. The distribution of these regions varies with time, as the superconductor moves in the external e.m. field, but for our reasoning we can consider the situation at a fixed instant. We can also suppose that the number of singular regions for unit volume remains almost constant until the external conditions are modified (rotation frequency, field parameters, temperature). The size of the critical regions is of the order of the coherence length ξ, that is, of the scale at which typically the variations of the order parameter take place in the given material.

As explained in Section 1, let us suppose that inside the singular regions the gravitational field is "forced" by the interaction with the condensate to oscillate around zero. We want to see how this can influence a pre-existing field configuration. Let us then consider, as done in Section 1.7, the functional integral of Euclidean gravity and add to the action a static cosmological source term $\mu^2(\mathbf{x})$. Let us also add a static source $T^{\mu\nu}(\mathbf{x})$ which generates a constant background field of strength g_E. We can write the averaged Euclidean field in weak field approximation as follows:

$$\langle 0|h_{\mu\nu}(\mathbf{x})|0\rangle = \frac{1}{Z}\int d[h]\, h_{\mu\nu}(x) \exp\left\{-\hbar^{-1}\left[\int d^4x\,\sqrt{g}\left(\frac{\Lambda}{8\pi G}+\frac{1}{2}\mu^2-\frac{R}{8\pi G}+\kappa h_{\mu\nu}T^{\mu\nu}\right)\right]\right\}. \tag{24}$$

where g and R must be expressed in terms of $h_{\mu\nu}$. The functional average is dominated by the functions h which minimize the action. But we know from the analysis of Section 1 that these functions are those which solve the field equation in the presence of the source $T^{\mu\nu}$ and of the constraints corresponding to the "saturation" of $|h|^2$ in the critical regions (7). Thus h is zero on the average in the critical regions. Since μ^2 and $T^{\mu\nu}$ do not depend on time, we expect that the average field does not depend on time either. It follows that the analytical continuation to imaginary time necessary to translate back the expectation value into Minkowski space is trivial.

3.2 Constrained field equation and its solution.

In order to guess the solution of the static field equation in the presence of the given source and constraints, let us now follow an analogy with an electrostatic field. In that case the regions in which the electric field and potential are forced to zero could be realized by very small perfect conducting grounded spheres or plaquettes.

We must check that the gravitational field equation in the case we are considering is analogous to that of the electrostatic field. We first write the equation of the trajectory $x^\alpha(\tau)$ of a particle in free fall in a given gravitational field $g_{\mu\nu}(x)$, called the geodesic equation:

$$\frac{d^2x^\alpha(\tau)}{d\tau^2} + \Gamma^\alpha_{\mu\nu}[x(\tau)]\frac{dx^\mu(\tau)}{d\tau}\frac{dx^\nu(\tau)}{d\tau} = 0, \tag{25}$$

where τ is the proper time, with differential $d\tau = \sqrt{dx^\mu dx^\nu g_{\mu\nu}}$ and $\Gamma^\alpha_{\mu\nu}$ is the Christoffel connection

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}\left(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu}\right). \tag{26}$$

In this equation and in the following we omit for simplicity the arguments of the fields.

We now specialize to the static case, supposing the particle initially at rest, and compute its acceleration. For the spatial components x^i eq. (25) takes the form

$$\frac{d^2x^i}{d\tau^2} + \Gamma^i_{00}\left(\frac{dx^0}{d\tau}\right)^2 = 0. \tag{27}$$

In the following we work in the weak field approximation and consider only terms of lowest order in κ. For the connection (26) we have

$$\Gamma^{(1)\,\alpha}_{\mu\nu} = \frac{1}{2}\kappa\delta^{\alpha\beta}\left(\partial_\mu h_{\beta\nu} + \partial_\nu h_{\beta\mu} - \partial_\beta h_{\mu\nu}\right) \tag{28}$$

and in particular, in the static case

$$\Gamma^{(1)\,i}_{00} = -\frac{1}{2}\kappa\partial^i h_{00}. \tag{29}$$

In eq. (27) the proper time τ differs from the coordinate time only for terms of order κ, thus to lowest order the acceleration is given by

$$\frac{d^2x^i}{dt^2} = G^i, \qquad \text{with } G^i = -\Gamma^{(1)\,i}_{00} = \frac{1}{2}\kappa\partial^i h_{00}. \tag{30}$$

On the other hand the field must satisfy Einstein vacuum equations $R_{\mu\nu} = 0$. Disregarding the quadratic terms in the connection we can write them as

$$\partial_\nu \Gamma^\rho_{\mu\rho} - \partial_\rho \Gamma^\rho_{\mu\nu} = 0. \tag{31}$$

For the 00 component in the static case one has

$$\partial_i \Gamma^i_{00} = 0. \tag{32}$$

In conclusion, the gravitational acceleration of a particle at rest is given to lowest order in κ by a vectorial field G^i with zero divergence in the vacuum which is the gradient of the potential $-\frac{1}{2}\kappa h_{00}$. This justifies, in that approximation, an analogy with the electrostatic field and the electrostatic potential.

Thus we can represent also the gravitational field in this case with field lines. Still following the electrostatic analogy (justified by the fact that the field equations and the boundary conditions for the two fields are the same), we observe that when a field line meets a singular region it is intercepted. The field lines are just conventional objects and the number of lines which cross the unity surface is proportional to the field intensity, the proportionality constant being arbitrary. Thus the final effect must not depend on such constant, as can be easily verified.

The fraction of intercepted field lines, corresponding to the shielding factor α, is approximately equal to the ratio between the total cross section of the singular regions and the area of the disk. Let us suppose that in the presence of shielding the gravity acceleration over the disk is $g_E(1-\alpha)$; we have in this simplified model

$$\alpha \sim \frac{N\sigma}{S_{disk}} = n\sigma, \tag{33}$$

where σ is the average cross section of a singular region, N is the total number of singular regions in the disk and n is the number of singular regions for unit disk surface.

3.3 Discussion.

The average value for $h_{\mu\nu}(\mathbf{x})$ obtained in the previous Section is unsatisfactory for two reasons: (1) it does not account for the observed non conservative character of the field in the presence of shielding (compare 2.1); (2) it does not reproduce the observed null effect below the disk.

In other words, while the observations indicate that there is a kind of "absorption" of the gravitational field in the disk, with a long cylindrical shielding region above it and no effect below, the field solution mentioned above has the typical features of an electric shielding: namely, a grounded conducting plaquette placed across a constant electrostatic field projects a "+/- shadow" in the field with a length of the same order of its width. (In the electric case, this is due to the superposition of the constant field and of the field produced by the electric charges induced on the plaquette.)

In fact we cannot be sure that averaging h as done above is correct. We know that in the quantum theory the gravitational force between two masses at rest is given in principle by eq. (23) but that equation does not give us enough information to compute the average field, nor it ensures that in the quantum case an average field is well defined at all. For instance, we would not be able to predict through eq. (23) if the gravitational red-shift is affected by the shielding. (This would actually be an interesting experimental check.)

Thus an important task for the theory would be that of evaluating eq. (23) in a suitable approximation in order to check if the resulting *effective* field corresponds to the observed configuration. At present we shall limit ourselves to the following consideration. We observe that in practice only the gravitational acceleration of the samples is measured in the experiment, i.e., the connection $\Gamma^{(1)\,i}_{00}$ (compare (30). We can admit that due to the quantum instability effect Γ vanishes in the critical regions. This perturbs slightly the pre-existing field configuration and produces a cylindrical shadow as observed. One can easily verify that a Γ-configuration of this kind still satisfies eq. (32) outside the disk and thus Einstein equations to lowest order. The energetic balance is ensured by the external non-gravitational source which constrains Γ (compare also the next Section).

4 Energetic balance.

After having introduced in the preceding Section a model which describes in an approximate way the variation of the gravitational field in the presence of the disk, it is necessary now to discuss some issues of elementary character, but important from the practical point of view, concerning the overall energetic balance of the shielding process.

In general one will have to supply energy in order to reduce the weight of an object, because the potential gravitational energy of the object has negative sign and is smaller, in absolute value, in the presence of shielding. Nevertheless, since the field is not conservative, it is certainly wrong to compute the difference in the potential energy of an object between the interior and the exterior of the shielding cone by evaluating naively the difference (which turns out to be huge) between an hypothetical "internal potential"

$$U = -\frac{GMM_E(1-\alpha)}{R_E} = -Mg_ER_E(1-\alpha), \tag{34}$$

where M is the mass of the object, R_E the Earth radius and M_E the Earth mass, and an "external potential" $U = -Mg_ER_E$.

Moreover, the gravitational fields with which we are most familiar, being produced by very large masses, are relatively insensitive to the presence of light test bodies and thus it makes sense in that case to speak of a field in the usual meaning: while a body falls down, we do not usually need to worry about its reaction on the Earth. On the contrary, in the present case the interaction between the shielded object and the external source (that is, the system [Bose condensate+external e.m. field]) which by fixing the constraints on the gravitational potential h produces the shielding, is very important. ‖

Let us then ask one "provocative" question, suggested by the experimental reality: if the superconducting disk is in a room and the shielding effect extends up to the ceiling, should we expect that the disk and all the shielding apparatus feel a back reaction? (And possibly an even stronger one if the ceiling is quite thick or if there are more floors above?) The most reasonable answer is, that since the ceiling is very rigid, the experimental apparatus is not able to exert any work on it and thus does not feel its presence.

To further clarify this point, suppose that we hang over the superconducting disk, before the shielding is produced, a spring of elastic constant k, holding a body of mass M at rest. Then we operate on the disk with proper e.m. fields and produce the shielding effect with

‖ Because of this, also considerations involving the energy density of the gravitational field, which can be properly defined for weak fields, are not helpful in the present case.

factor α, that means, the gravity acceleration over the disk becomes $g_E(1-\alpha)$. If the shielding effect is obtained quickly, the mass will begin to oscillate, otherwise it will rise by an height $\Delta x = \alpha g_E M k^{-1}$, while remaining in equilibrium. In any case, since for a harmonic oscillator in motion the kinetic energy and the potential energy have the same mean value, it is legitimate to conclude that the shielding apparatus has done on the system [mass+spring] a work of the order of

$$\Delta E \sim k(\Delta x)^2 \sim (\alpha g_E M)^2 k^{-1}. \tag{35}$$

This example shows that the work exerted by the apparatus on a sample in order to "shield it" will depend in general of the response of the sample itself, being larger when such response is large itself.

At this point we can estimate how much energy is needed in this case to bring over the critical density one region of the condensate of cross section σ (compare eq. (33)). If σ_{sample} is the projection of the sample on the disk, this energy is given by

$$\Delta E_\sigma = \frac{\sigma}{\sigma_{sample}} \Delta E \sim \frac{\sigma}{\sigma_{sample}} (\alpha g_E M)^2 k^{-1}. \tag{36}$$

This energy must be supplied by the external variable e.m. field.

In conclusion, we must expect in general an interaction between the partially shielded samples and the shielding apparatus. The energy needed to shield a sample depends on the mass of the sample itself and on the way it is constrained to move. In particular, we deduce from eq. (35) that if we want to detect the shielding effect by measuring the deformation of a spring, in order to do this with the smallest influence on the shielding apparatus we should use, as far as allowed by the sensitivity of the transducer, a spring with high rigidity coefficient k.

5 The "threshold" hypothesis.

5.1 Estimate for $\mu^2(x)$.

A local cosmological term can induce gravitational instabilities in those regions where its total sign is positive. We have shown that the contribution of a Bose condensate to the cosmological term is positive (in Euclidean spacetime) and equal to $\frac{1}{2}\mu^2(x) = \frac{1}{2}[\partial_\mu \phi_0^*(x)][\partial^\mu \phi_0(x)] +$

$\frac{1}{2}m_\phi^2|\phi_0(x)|^2$ (see eq.s (3), (4)). It is important to give a numerical estimate of the magnitude order of this contribution in the case of a superconductor.

To this end we recall that the Hamiltonian of a scalar field ϕ of mass m_ϕ is given by

$$H = \frac{1}{2}\int d^3x \left\{ \left| \frac{\partial\phi(x)}{\partial t} \right|^2 + \sum_{i=1}^{3} \left| \frac{\partial\phi(x)}{\partial x^i} \right|^2 + m_\phi^2|\phi(x)|^2 \right\}. \tag{37}$$

In our case ϕ describes a system with a condensate and its vacuum expectation value is $\langle 0|\phi(x)|0\rangle = \phi_0(x)$. We then have

$$\langle 0|H|0\rangle = \frac{1}{2}\int d^3x\mu^2(x). \tag{38}$$

In the non-relativistic limit, appropriate in our case, the energy of the ground state is essentially given by $\mathcal{N}Vm_\phi$, where m_ϕ is the mass of a Cooper pair (of the order of the electron mass; in natural units $m_\phi \sim 10^{10}\ cm^{-1}$), V is a normalization volume and $\mathcal{N}$ is the number of Cooper pairs for unit volume. Assuming $\mathcal{N} \sim 10^{20}\ cm^{-3}$ at least we obtain

$$\mu^2 \sim \mathcal{N}m_\phi > 10^{30}\ cm^{-4} \qquad \text{(In a superconductor.)} \tag{39}$$

We also find in this limit $|\phi_0| \sim \mathcal{N}/\sqrt{m_\phi}$.

As we saw in Section 1, a typical upper limit on the intrinsic cosmological constant observed at astronomical scale is $|\Lambda|G < 10^{-120}$, which means $|\Lambda|/8\pi G < 10^{12}\ cm^{-4}$. This small value, compared with the above estimate for $\mu^2(x)$, supports our hypothesis that the total cosmological term can assume positive values in the superconductor and the criticality condition can be satisfied in some regions. But in fact the positive contribution of the condensate is so large that one could expect the formation of gravitational instabilities in any superconductor, subjected to external e.m. fields or not – a conclusion which contrasts with the observations.

We wonder if the value of $\Lambda/8\pi G$ at small scale could be larger than that observed at astronomical scale and negative in sign, in such a way to represent a "threshold" of the order of $\sim 10^{30}\ cm^{-4}$ for anomalous gravitational couplings. As we recalled in Section 1, a negative intrinsic cosmological constant can be present in models of quantum gravity containing a fundamental length. With a magnitude as mentioned above, it would not affect any other known physical process.

5.2 Threshold versus power transfer efficiency.

In our opinion the hypothesis of a threshold accounts quite well for the features of the observed effect and in fact we have implicitly accepted it in several qualitative points of our analysis, especially in Section 2. As we pointed out throughout the paper, all evidences show that a proper "driving" and forcing of the supercurrents in the disk and thus of the Cooper pairs density are essential in order to obtain the shielding effect and to improve it.

However, the available experimental data are not sufficient yet to decide whether the hypothesis of a threshold for the Cooper pairs density is necessary, or it is only a helpful schematic representation. In fact, as we stressed in Section 4, a global energy balance must be respected in the shielding process. This energetic requirement might be very important in determining the critical regions.

Let us consider for instance another system in which a Bose condensate, described by a macroscopic wavefunction, is present: superfluid helium. From eq. (39) we see that in that case μ^2 is ca. 10^3 times larger than in an electronic Bose condensate. But superfluid helium does not show, according to common knowledge (although specific data are not available), any anomalous gravitational effects. To explain this we can observe that unlike an electronic condensate, superfluid helium is neutral and thus cannot absorb energy from an external e.m. field. Moreover, its specific heat is so low, that in general any power transfer process would be severely constrained. It is thus possible in our opinion that although its density is much larger than that of an electronic condensate, superfluid helium does not cause any appreciable shielding effect, since a modification of the field $h_{\mu\nu}(x)$ as described in Section 3, with ensuing reduction of the samples' weight, would not be sustained energetically. In other words, the local "saturation" of the field $h_{\mu\nu}(x)$ represents an energetically less favoured state and in the case of superfluid helium there is no suitable external energy source to allow the transition to this state.

Summarizing, the following reasoning holds, at least qualitatively, and might be applied to the next available experimental data. We have seen that the shielding phenomenon involves a power transfer process, which in general can be more or less efficient. There are two limiting cases:

1. The power transfer is very efficient. Then the shielding factor α is fixed by the number of critical regions and by their average cross section (compare eq. (33)), independently of the energy transferred to the samples.

2. The power transfer is inefficient. In this case the shielding factor α can depend on the energy transferred to the samples. Thus α can depend on the mass of the samples. For samples with the same mass, it can depend on their cross section and possibly on the "rigidity" parameter k (compare eq. (36)).

Suppose now to be in the limiting case (1), as experimentally it appears to be the case, at least for samples whose mass does not exceed $\sim 100\ g$. If in these conditions the shielding factor α depends strongly on the rotation speed of the disk, on the applied e.m. field and in general on those factors which imply the generation of density variations in the condensate, this means that the number of critical regions and their average cross section depend strongly on such variations. In turn, the latter means that the existence of a threshold is very likely.

6 Concluding remarks.

The analysis of Section 1 allows us to conclude that there is broad evidence for instability of the quadratic part of the Euclidean gravitational action in the presence of a Bose condensate. We recall our starting hypotheses:

1. Validity of the Euclidean formalism in the context of weak-field approximation was assumed. As discussed in Section 1.3 there are no reasons to doubt of such validity in our case.

2. We admitted (compare also Section 1.7) that the vacuum expectation value $\phi_0(x)$ of the bosonic field is determined by external factors: e.m. field, temperature, microscopic structure of the material etc. From a phenomenological point of view this approach is completely justified and allows to divide the problem in two parts: the dynamics of the

source $\phi_0(x)$ and the effects of the source on the gravitational field. This approximation is not adequate when the back-reaction of the gravitational field on the source cannot be disregarded (compare Section 4).

The description of the effects of the instability in terms of a classical "modified field" (Section 3) is physically helpful, even though it leads to only partially correct consequences.

In general, in the energetic balance for heavy samples one should keep into account the back-reaction mentioned above.

The existence of a critical threshold for the value of the induced cosmological term $\mu^2(x)$ is theoretically appealing and in reasonable agreement with the experimental observations.

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